Mark Scheme (Results)

January 2023

Pearson Edexcel International Advanced Level
In Pure Mathematics P2 (WMA12) Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for this paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

'M' marks
These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.
e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.
The following criteria are usually applied to the equation.

To earn the M mark, the equation
(i) should have the correct number of terms
(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct
e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.
For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.
e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

'A' marks
These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. e.g. M0 A1 is impossible.

'B' marks
These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph).

A few of the A and B marks may be f.t. – follow through – marks.

3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC – special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp – decimal places
- sf – significant figures
- $\ast$ – The answer is printed on the paper
- $\llcorner$ – The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao), unless shown, for example as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Pure Mathematics Marking
(But note that specific mark schemes may sometimes override these general priniciples)

Method mark for solving 3 term quadratic:

1. **Factorisation**
   $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \ldots$
   $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \ldots$
2. **Formula**
   Attempt to use the correct formula (with values for *a*, *b* and *c*).
3. **Completing the square**
   Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, q \neq 0$, leading to $x = \ldots$

Method mark for differentiation and integration:

1. **Differentiation**
   Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. **Integration**
   Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula
Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values. Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers
Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working
The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1(a)	$h=0.5$	B1
	$A\approx\frac{1}{2}\times\frac{1}{2}\left\{2.287+2.834+2\left(4.470+6.719+7.291\right)\right\}$	M1
	= awrt 10.52	A1
		(3)
(b)(i)	$A\approx"10.52"-4$	M1
	= 6.52	A1ft
(b)(ii)	$A\approx"10.52"$	B1ft
		(3)
		Total 6

(a)

B1 $h=0.5$ oe seen or implied by sight of $\frac{0.5}{2}$ in front of the bracket. May also be implied by a correct answer if no incorrect working seen. $h=-0.5$ is B0

M1 Correct application of the trapezium rule with their *h*. Look for a correct bracket structure, condoning slips copying values from the table or the omission of the final brackets on the rhs

e.g. $\frac{1}{2}\times"\frac{1}{2}"(2.287+2.834+2(4.470+6.719+7.291$ is M1

but $\frac{1}{2}\times"\frac{1}{2}"\times 2.287+2.834+2(4.470+6.719+7.291)$ is M0 unless the brackets are recovered or implied by their answer (you may need to check this).

Also allow for a correct method adding individual trapezia using their *h* condoning copying errors but the brackets must be correct (or recovered or implied by later work)

e.g. $\frac{1}{2}\times"\frac{1}{2}"(2.287+4.470)+\frac{1}{2}\times"\frac{1}{2}"(4.470+6.719)+\frac{1}{2}\times"\frac{1}{2}"(6.719+7.291)+\frac{1}{2}\times"\frac{1}{2}"(7.291+2.834)$

A1 awrt 10.52 isw once a correct answer is seen. Correct answer with no working scores B1M1A1 but if there is evidence of using $h=-0.5$ then maximum awarded is B0M1A0

(b)(i) Note that they must be using their answer to part (a) to score marks in (b)

M1 Attempts to subtract 4 from their answer to part (a) or allow for the expression $"10.52"-\left[2x\right]_{-1}^{1}$ or e.g. $"10.52"-(2+2)$. Do not condone omission of brackets or arithmetical errors here. This mark cannot be scored from a new attempt at the trapezium rule.

$"10.52"-(2-2)$ is M0

A1ft awrt 6.52 correct answer or correct ft (allowing if they proceed to round a more accurate answer in (a) to 2dp). In both cases they must be using their answer to part (a) function which could even come from integration.

(ii)

B1ft awrt 10.52 Writes down the same answer as part (a) (follow through their (a)). Condone "same as **part (a)**" (again allow if they proceed to round a more accurate answer in (a) to 2dp)

Note (a) 10.5 (b)(i) 6.5 (b)(ii) 10.5 could score B1M1A0M1A1ftB1ft

Question Number	Scheme	Marks
2(a)	$(S=)\ 6x^2+6xh+2xh$	B1
	eg $V=3x^2h=972\Rightarrow h=\dfrac{972}{3x^2}$ or eg $hx=\dfrac{324}{x}$ $\Rightarrow (S=)\ 6x^2+8x\left(\dfrac{972}{3x^2}\right)$ or $\Rightarrow (S=)\ 6x^2+8\left(\dfrac{324}{x}\right)$	M1
	$S=6x^2+\dfrac{2592}{x}$ *	A1*
		(3)
(b)	$\left(\dfrac{\mathrm{d}S}{\mathrm{d}x}=\right)12x-\dfrac{2592}{x^2}$	B1
		(1)
(c)	$12x-\dfrac{2592}{x^2}=0\Rightarrow 12x^3=2592$ $\Rightarrow x=\sqrt[3]{\dfrac{2592}{12}}$	M1
	$x=6$	A1
		(2)
(d)	$\left(\dfrac{\mathrm{d}^2S}{\mathrm{d}x^2}=\right)12+\dfrac{5184}{x^3}$	B1ft
	$\dfrac{\mathrm{d}^2S}{\mathrm{d}x^2}>0$ when $x=6$ so minimum	B1
		(2)
(e)	$S=6(6)^2+\dfrac{2592}{6}=648\ (\text{cm}^2)$	B1
		(1)
		Total 9

(a)

B1 Correct expression for the surface area in any form.
It may be implied by a correct expression for the surface area in terms of x which is not the given answer.
Once a correct expression is seen then award B1.

M1 Uses the given volume in a dimensionally correct formula to obtain h or hx in terms of x and substitutes into their expression for S of the form $...xh+...x^2$ or equivalent e.g. $...xh+...xh+...x^2$
It may be implied by a correct expression for the surface area in terms of x which is not the given answer.

A1* Achieves the given answer with no errors including bracketing omissions.
As a minimum you must see:

- the separate equation for the volume
- a substitution before seeing the $\dfrac{2592}{x}$ term
- $S=$ in their solution at some point (or e.g. SA = or surface area = or Area=)

Examples:

e.g. $6x^2+6x\left(\frac{324}{x^2}\right)+2x\left(\frac{324}{x^2}\right)$ scores B1M1A0

e.g. $6x^2+8x\left(\frac{972}{3x^2}\right)\Rightarrow S=6x^2+\frac{2592}{x}$ scores B1M1A0 (does not write a separate equation for the volume)

e.g. $$3x^2h=972\Rightarrow S=6x^2+6x\left(\frac{324}{x^2}\right)+2x\left(\frac{324}{x^2}\right)$$
$$=6x^2+\frac{2592}{x}$$

scores B1M1A1 (separate equation for the volume, a substitution before seeing the $\frac{2592}{x}$ term and S = seen in their solution at some point.

(b) Mark (b), (c), (d) and (e) altogether

B1 $\left(\frac{\mathrm{d}S}{\mathrm{d}x}=\right)12x-\frac{2592}{x^2}$ in any form eg $12x-2592x^{-2}$

(c)

M1 Starts from a derivative of the form $Ax-Bx^{-2}$ oe, sets equal to 0 (seen or implied) where $A\times B>0$, and solves via a correct method to find an expression or value for $x=\sqrt[3]{\frac{B}{A}}$ (you may need to check this)

$\sqrt[3]{\frac{2592}{12}}$ scores M1.

Stating 6 without a **correct derivative from (b)** is M0.

M1 6 cao with no incorrect working seen in their method. Withhold this mark if ± 6 seen

Minimum acceptable for full marks is $12x-\frac{2592}{x^2}=0\Rightarrow x=6$

Note this final mark is only possible from a correct derivative

(Beware of incorrect derivatives leading to the correct value of x e.g. $6x-\frac{1296}{x^2}=0\Rightarrow x=6$)

(d)

B1ft Correct second derivative. Follow through their first derivative of the form $Ax-Bx^{-2}$ so award for $A+\frac{2B}{x^3}$ or equivalent.

B1 Fully correct justification with a conclusion (or preamble with eg tick/QED etc) following fully correct work to find $\frac{\mathrm{d}^2S}{\mathrm{d}x^2}$. They must either:

- substitute $x=6$ into $12+\frac{5184}{x^3}$ (if evaluated achieving 36) and state e.g. $(36)>0$ oe **hence minimum**

- justifiy that as $x > 0$ or $x = 6$ then $12 + \dfrac{5184}{x^3} > 0$ (or $\dfrac{d^2S}{dx^2} > 0$) **hence minimum**.

Note this final mark is only possible if $x = 6$ is found correctly in (c) from a correct derivative.

(e)

B1 648 cm^2 (condone lack of units/incorrect units) provided this has come from $x = 6$

Note e.g. $x = -12$ leading to 648 is B0

Question Number	Scheme	Marks
3(a)	$\left(2+\frac{kx}{8}\right)^7 = 2^7 + \binom{7}{1}2^6\left(\frac{kx}{8}\right) + \binom{7}{2}2^5\left(\frac{kx}{8}\right)^2 + \binom{7}{3}2^4\left(\frac{kx}{8}\right)^3 + ...$	M1
	$= 128 + 56kx + \frac{21}{2}k^2x^2 + \frac{35}{32}k^3x^3 + ...$	B1A1A1
		(4)
(b)	$\frac{35}{32}k^3 - \frac{21}{2}k^2 = \frac{21}{2}k^2 - 56k$	M1
	$5k^2 - 96k + 256 = 0 \Rightarrow k = ...$	dM1
	$k = 16, \frac{16}{5}$	A1
		(3)
		Total 7

(a)

M1 Attempts the binomial expansion **up to at least the third term** with an acceptable structure for either the **3rd or 4th term**. The correct binomial coefficient (allowing alternative notation) must be combined with the correct power of x and the correct power of 2 (oe) but condone if brackets are missing. Condone errors on the first or second term.

M0 for descending powers.

Alternatively writes e.g. $\left(2+\frac{kx}{8}\right)^7 = 2^7\left(1+\frac{kx}{16}\right)^7 = 2^7\left(1+\frac{7}{16}kx+\frac{7\times6}{2}\left(\frac{kx}{16}\right)^2+\frac{7\times6\times5}{6}\left(\frac{kx}{16}\right)^3\right)$ which can also score M1 for the expansion up to at least the third term with an acceptable structure for either the **3rd or 4th term**. They do not have to multiply out the brackets for this mark but the 2^7 cannot be omitted unless it is later recovered. Condone errors on the first or second term.

Condone missing brackets.

B1 $128+56kx$. May be listed but must be simplified. Allow $128\left(1+\frac{7}{16}kx+...\right)$ if the 2^7 is taken out first.

A1 $\frac{21}{2}k^2x^2$ **or** $\frac{35}{32}k^3x^3$. May be listed, **does not need to be simplified**, but the binomial coefficients must be numerical. Accept $\frac{21}{2}(kx)^2$ **or** $\frac{35}{32}(kx)^3$ oe e.g. $10.5(kx)^2$ or $1.09375(kx)^3$ for this mark only. Common acceptable terms seen are $\frac{672}{64}(kx)^2$, $\frac{560}{512}(kx)^3$

A1 $\frac{21}{2}k^2x^2$ **and** $\frac{35}{32}k^3x^3$ or exact simplified equivalent e.g. $10.5k^2x^2$ **and** $1.09375k^3x^3$. May be listed but must have k^nx^n terms.

Note: isw after correct terms are seen if they try to divide or multiply through or set = 0

(b)

M1 Uses the given information to usually form a cubic equation in k only (or quadratic if they have cancelled a k from each term) using their coefficients from (a) in the correct positions.

e.g. $"\frac{35}{32}k^3" - "\frac{21}{2}k^2" = "\frac{21}{2}k^2" - "56k"$ or $"\frac{35}{32}k^3" + "56k" = 2"\frac{21}{2}k^2"$

Implied by $5k^2 - 96k + 256 = 0$ or equivalent. (The = 0 may be implied by later work)

e.g. $35k^2 - 672k + 1792 = 0$ or $\frac{35}{32}k^2 - 21k + 56 = 0$

Their equation should be able to be simplified to a 3TQ=0

Condone coefficients appearing as rounded decimals.

dM1 Achieves a 3 term cubic or a 3 term quadratic and solves (which may be directly via a calculator) to find a value of k (which may be unsimplified). Usual rules apply for solving a quadratic. Do not be concerned by their rearrangement and the = 0 may be implied.

Only allow real solutions.

It is dependent on the first method mark.

A1 16, $\frac{16}{5}$ oe If $k = 0$ is stated then it must be rejected.

Do not isw if they proceed to a range of values e.g. $\frac{16}{5} < k < 16$

Question Number	Scheme	Marks
4(i)	E.g. $2=\log_3 9$ Eg $\log_3(4x)-\log_3(5x+7)=\log_3\dfrac{4x}{5x+7}$	M1
	E.g. $36x=5x+7$ $\dfrac{4x}{5x+7}=\dfrac{1}{9}$	A1
	$x=\dfrac{7}{31}$	A1
		(3)
(ii)	$\left(\sum_{r=1}^{2}\log_a y^r=\right)\log_a y+\log_a y^2$ or $\left(\sum_{r=1}^{2}(\log_a y)^r=\right)\log_a y+(\log_a y)^2$	B1
	$\log_a y+\log_a y^2=\log_a y+(\log_a y)^2\Rightarrow 2\log_a y-(\log_a y)^2=0$ $\Rightarrow \log_a y(2-\log_a y)=0\Rightarrow \log_a y=2$	M1
	$y=a^2$	A1
		(3)
		Total 6

(i) Answer on its own with no working score 0 marks.

M1 Demonstrates at least one correct law of logarithms.

e.g. $2=\log_3 9$ or $\log_3(4x)=\log_3 4+\log_3 x$ or $\log_3(4x)-\log_3(5x+7)=\log_3\dfrac{4x}{5x+7}$

$\log_3(5x+7)-\log_3(4x)=\log_3\dfrac{5x+7}{4x}$

(Do not be concerned with slips on the 2 if manipulating the equation

e.g. $\log_3(4x)+2=\log_3(5x+7)\Rightarrow\log_3\dfrac{4x}{5x+7}=2$ scores M1)

A1 For a correct equation in any form with no logs eg $4x=\dfrac{5x+7}{9}$. Must have come from correct log work.

A1 $\dfrac{7}{31}$ only and follows a correct equation with no logs.

Note $\dfrac{\log_3 5x+7}{\log_3 4x}=-2\Rightarrow x=\dfrac{7}{31}$ will only score a maximum of M1A0A0 if they are able to demonstrate at least one correct law of logarithms.

(ii) Answer on its own with no working score 0 marks.

B1 For either (or both) summation correct $\log_a y+\log_a y^2$ or $\log_a y+(\log_a y)^2$ or equivalent

Can also be awarded for a correct equation:

e.g. $\log_a y^2=(\log_a y)^2$ or $2\log_a y=(\log_a y)^2$ or $3\log_a y=\log_a y+(\log_a y)^2$

Sight of either expression or the equation scores even if they make subsequent errors.
Poor bracketing may be recovered or implied by later work. Do not penalise the absence of base *a* on some or all of the log terms for this mark.

M1 Proceeds from their quadratic equation in $\log_a y$, collects terms, factorises or cancels $\log_a y$ and obtains $\log_a y = k$. They may define $\log_a y$ in terms of another variable e.g. *x* such that $x + 2x = x + x^2 \Rightarrow x = k$ which is acceptable
Alternatively, they may change the base and obtain $\log_a y = k$
e.g. $\log_a y^2 = (\log_a y)^2 \Rightarrow \frac{\log_a y^2}{\log_a y} = \log_a y \Rightarrow \log_y y^2 = \log_a y \Rightarrow \log_a y = 2$

Question Number	Scheme	Marks
5(a)	$3^3+3^2\left(p+3\right)-3+q=0$	M1
	eg $\Rightarrow 27+9p+27-3+q=0 \Rightarrow 9p+q=-51$*	A1*
		(2)
(b)	$\left(-p\right)^3+\left(p+3\right)\left(-p\right)^2-\left(-p\right)+q=9$	M1
	eg $-p^3+p^3+3p^2+p+q=9 \Rightarrow 3p^2+p+q-9=0$*	A1*
		(2)
(c)	$3p^2+p+q-9=0$ $\Rightarrow 3p^2+p-51-9p-9=0$ $\Rightarrow 3p^2-8p-60=0$	M1
	$p=6$	A1
	$q=-51-9p=-105$	A1
		(3)
(d)	$\mathrm{f}\left(x\right)=x^3+9x^2-x-105$ $\mathrm{f}\left(x\right)=\left(x-3\right)\left(...x^2+...x+...\right)$	M1
	$\mathrm{g}\left(x\right)=x^2+12x+35$	A1
		(2)
		Total 9

(a)

M1 Attempts to use the factor theorem by setting $\mathrm{f}(\pm 3)=0$. Score for the values embedded in the expression leading to an equation in *p* and *q*. The = 0 may be implied by later work for this mark.

Alternatively, there may be other attempts eg to divide algebraically by $x-3$

$$\begin{array}{r l}
 & x^2+(p+6)x+3p+17 \\
x-3 & \overline{)\,x^3+(p+3)x^2 \qquad -x \qquad +q} \\
 & x^3 \qquad -3x^2 \\
 & \qquad (p+6)x^2 \qquad -x \\
 & \qquad (p+6)x^2+(-3p-18)x \\
 & \qquad\qquad (3p+17)x \qquad +q \\
 & \qquad\qquad (3p+17)x-9p-51
\end{array}$$

$$\Rightarrow -9p-51=q \Rightarrow 9p+q=-51$$

To score the method mark they would need to proceed as far as equating *q* with their $"-9p-51"$ to achieve an equation in *p* and *q*.

With alternative methods look for a correct method, condoning slips and invisible brackets leading to an equation in *p* and *q*.

A1* Correct proof with no errors including brackets. There must be at least one intermediate stage of working and $=0$ must be seen at some point in their solution via the factor theorem method.

e.g. $3^3+(p+3)\times 3^2-3+q=0 \Rightarrow 9p+q=-51$ scores M1A0 (no intermediate stage seen)

$27+(p+3)\times 9-3+q=0 \Rightarrow 51+9p+q=0 \Rightarrow 9p+q=-51$ scores M1A1

(b)

M1 Attempts the remainder theorem by setting $\mathrm{f}(\pm p)=9$ oe Award for e.g. $(-p)^3+(p+3)(-p)^2+p+q=9$ condoning sign slips and invisible brackets.

Alternatively, they may attempt to divide algebraically by $x+p$ leading to a remainder in terms of p^2, p and q which is equated to 9. Condone slips in their working.

$$
\begin{array}{r|llll}
 & x^2 & +3x & -1-3p & \\
\hline
x+p & x^3+(p+3)x^2 & -x & +q & \\
 & x^3 \quad px^2 & & & \\
 & 3x^2 & -x & & \\
 & 3x^2 & +3px & & \\
 & & (-1-3p)x & +q & \\
 & & (-1-3p)x+(-1-3p)p & & \\
\end{array}
$$

With alternative methods look for a correct method, leading to a remainder in terms of p^2, p and q which is equated to 9

A1* Correct proof with no errors including brackets. There must be at least one intermediate stage of working before proceeding to the final answer

e.g. $(-p)^3+(p+3)(-p)^2-(-p)+q=9 \Rightarrow 3p^2+p+q-9=0$ scores M1A0 (no intermediate stage)

$(-p)^3+(p+3)(-p)^2-(-p)+q=9 \Rightarrow 3p^2+p+q=9 \Rightarrow 3p^2+p+q-9=0$ scores M1A1

(c)

M1 Attempts to use both given equations to form a 3TQ equation in *p* (or *q*). (terms do not need to be all on one side and condone the omission of = 0)

Do not be too concerned by slips in their substitution/rearrangements or miscopying of the given equations. May be implied by either a correct value for *p* or a correct value for *q*

A1 $p=6$ (ignore any reference to $-\frac{10}{3}$)

A1 $q=-105$ only

(d)

M1 Uses their values for *p* and *q* and a correct strategy (inspection or long division) to obtain the quadratic factor.

Via inspection score for $x^2+...x\pm\frac{"q"}{3}$

Via long division score for proceeding as far as $x^2\pm"(p+6)"x$. If they attempt algebraic division in (a) you may need to check their quotient with their value for *p* to see if the method mark can be scored. They may also restart which is acceptable.

Condone the use of a negative value for *p* for this mark (eg even if $p=-6 \Rightarrow 0x$)

A1 $x^2+12x+35$ oe eg $(x+5)(x+7)$ Allow embedded as $(x-3)(x^2+12x+35)$ and condone poor notation such as $\mathrm{f}(x)=x^2+12x+35$. Also allow this mark to be scored if seen within their long division.

Question Number	Scheme	Marks
6(a)(i)	Centre is (–4, 2)	B1
(ii)	$x^2+y^2+8x-4y=0$ $\Rightarrow(x+4)^2+(y-2)^2-16-4=0\Rightarrow r=...$	M1
	$r=2\sqrt{5}$ oe	A1
		(3)
(b)	$x^2+y^2+8x-4y=0,\ x+2y+10=0$ $\Rightarrow(-2y-10)^2+y^2+8(-2y-10)-4y=0$ or $x^2+\left(\dfrac{-x-10}{2}\right)^2+8x-4\left(\dfrac{-x-10}{2}\right)=0$	M1
	$y^2+4y+4=0$ or e.g. $x^2+12x+36=0$	A1
	$(y+2)^2=0\Rightarrow y=...$ or e.g. $(x+6)^2=0\Rightarrow x=...$	dM1
	(–6, –2)	A1
		(4)
(c)	$x+2y+10=0\Rightarrow m_T=-\dfrac{1}{2}\Rightarrow m_N=2$ or $m_N=\dfrac{"2"-("-2")}{"-4"-("-6")}$	M1
	Usually either $y-2=2(x+4)$ or $y+2=2(x+6)$	dM1
	$y=2x+10$	A1
		(3)
		Total 10

(a)(i) Mark (i) and (ii) together.

B1 Correct centre. Allow written as $x=-4,\ y=2$

(ii)

M1 Correct method for the radius. Award for $(x\pm a)^2+(y\pm b)^2....\Rightarrow r=\sqrt{"a^2"+"b^2"}=...$ or maybe seen using the general equation of a circle e.g. $x^2+2gx+y^2+2fy+c=0\Rightarrow r=\sqrt{f^2+g^2-c}$
May be implied by a correct radius for their centre if no method is shown.

A1ft $r=2\sqrt{5}$ or exact equivalent eg $\sqrt{20}$. Do not accept $\pm$.
Only follow through their centre with coordinates $(\pm4,\pm2)$

(b) Note that answer only scores 0 marks.

M1 Uses both equations to eliminate one variable. Condone slips in substituting in. Also allow use of the equation of the circle in the form $(x-a)^2+(y-b)^2=r^2$ oe allowing for slips in manipulation

A1 Correct 3TQ or any multiple of this (allow any equivalent 3 term equations) which do not need all the terms on one side of an equation. Condone lack of = 0

dM1 Solves their 3TQ to obtain the *x* or *y* coordinate of *P*. Usual rules apply. (May need to check if via a calculator). It is dependent on the first method mark.

A1 Correct coordinates. Allow written as $x=-6,\ y=-2$.

Alt (b) Geometric approach example

M1 Forms a right angled triangle using the radius as the hypotenuse and letting the two shorter sides be 2*a* and *a* (gradient of the radius from the centre to tangent is 2) such that $(2a)^2+a^2=("2\sqrt{5}")^2 \Rightarrow a=...$

A1 $a=2$

dM1 A correct method to find *x* or *y*: $x="-4"-"2"=...$ and $y="2"-2("2")=...$

A1 $(-6, -2)$

Alt (b) Finding the point of intersection between the tangent and the normal

M1 Attempts to find the equation of the normal using their centre and a gradient of 2

A1 $y=2x+10$

dM1 Attempts to solve simultaneously to find *x* or *y*

A1 $(-6, -2)$

Alt (b) Implicit differentiation

M1 Attempts to differentiate implicitly, substituting in $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{2}$ and using the equation of the tangent to *C* at *P* to eliminate one variable. $x^2+y^2+8x-4y=0 \Rightarrow 2x+2y\frac{\mathrm{d}y}{\mathrm{d}x}+8-4\frac{\mathrm{d}y}{\mathrm{d}x}=0$

$\Rightarrow 2x+2y\left(-\frac{1}{2}\right)+8-4\left(-\frac{1}{2}\right)=0 \Rightarrow 2x-y+10=0$

$\Rightarrow 2(-2y-10)-y+10=0$ or $x+2(2x+10)+10=0$ (using the equation of the tangent to *C* at *P*)

A1 A correct simplified linear equation in *x* or *y*
e.g. $5y=-10$ or $5x=-30$ which may be implied by the correct *x* or *y* coordinate.

dM1 Attempts to solve their linear equation to find *x* or *y*

A1 $(-6, -2)$

There may be other credit worthy methods so look carefully at their solution and send to review if unsure

(c) Credit worthy work may be seen in (b) but must be used in (c)

M1 Attempts to find the gradient of the line between their centre and their *P* by either using

- the given tangent equation which may be implied by sight or use of gradient = 2
- their centre and their *P*. Score for a correct expression to find the gradient using their centre and their *P* eg $\frac{"2"-("-2")}{"-4"-("-6")}$. Also allow with other valid coordinates which would lie on the normal e.g. $(-2, 6)$ which may have come from solving simultaneously the equation of the normal with the equation of the circle.

Allow this mark if they then subsequently find the negative reciprocal of this.

dM1 Correct straight line method using their normal gradient (via a correct method) and their centre or their *P* (or any other valid pair of coordinates which would lie on the normal
e.g. using their gradient using a geometric approach 1 to the right, 2 up

If they use $\left(y - y_1\right) = m(x - x_1)$ then both brackets including signs must be correct for their coordinates.
If they use $y = mx + c$ they must proceed as far as $c = ...$
It is dependent on the previous method mark.

A1 $y = 2x + 10$ cao

Question Number	Scheme	Marks
7(a)	$r=\sqrt{\frac{12.8}{20}}=0.8*$ or e.g. $20\times0.8\times0.8=12.8$ so r = 0.8*	B1*
		(1)
(b)	$a=20\div0.8^2$	M1
	= 31.25	A1
		(2)
(c)	$\frac{31.25\left(1-0.8^n\right)}{1-0.8}>156$	M1
	Eg $1-0.8^n>0.9984$ $\Rightarrow 0.8^n<0.0016$	dM1
	$0.8^n<0.0016\Rightarrow n>\frac{\log\left(0.0016\right)}{\log(0.8)}$ or $0.8^n<0.0016\Rightarrow n>\log_{0.8}0.0016$	M1
	$n=29$	A1
		(4)
		Total 7

(a)

B1* Correctly demonstrates r = 0.8 oe with no incorrect working seen. If they use 0.8 within their solution there must be either some preamble or concludes $r=0.8$

Minimum acceptable is an expression for r which is not 0.8 or an equation involving r with all values substituted in. At some point we must see r being linked with 0.8 either as part of an equation or e.g. with a conclusion r = 0.8

May all be in fractions instead eg $20\times\frac{4}{5}\times\frac{4}{5}=\frac{64}{5}\Rightarrow r=\frac{4}{5}$ is B1

Note that $ar^5=12.8,\ ar^3=20\Rightarrow r=\sqrt{\frac{12.8}{20}}=0.8$ is B0

(b) Work seen in (a) must be used or stated in (b) to score.

M1 Correct method for the first term. Score for the expression $12.8\div0.8^4$ or $20\div0.8^2$ oe

May be seen as $\frac{64}{5}\div\left(\frac{4}{5}\right)^4$ (Do not penalise invisible brackets for this mark).

If a candidate has stated incorrectly $ar^5=12.8$ or $ar^3=20$ in (a) then allow M1 if they use this correctly to find a for their equation.

A1 31.25 or $\frac{125}{4}$ or $31\frac{1}{4}$ with no incorrect working seen

(c) Note on EPEN this is M1A1M1A1 but we are marking this M1M1M1A1

M1 Attempts to set up an equation or inequality using their first term and r = 0.8 in the sum formula.

Score for $\dfrac{"31.25"\left(1-0.8^n\right)}{1-0.8}...156$ (Allow any inequality or =)

May be seen as $\dfrac{"\dfrac{125}{4}"\left(1-\left(\dfrac{4}{5}\right)^n\right)}{1-\dfrac{4}{5}}...156$

dM1 Rearranges to $A\times 0.8^n...B$ where A could be 1 and B could be unsimplified (Allow any inequality or =).
Do not be concerned by the mechanics of their rearrangement. It is dependent on the first method mark.
May be implied by further work if logarithms are used correctly.

M1 Attempts to find n by solving 0.8^n...their 0.0016 **using logarithms** correctly. (May appear as $0.8^n...\text{their}\,\dfrac{1}{625}$ or even $0.8^{-n}...\text{their}\,625$. Cannot be scored from an unsolvable equation.
(Allow any inequality or =).
This can also be scored for solving an equation or inequality involving a term rather than the sum such as $0.8^{n-1}=...$

A1 29 cso (It must have come from a correct equation (although allow any inequality)).
All three Ms must have been earned.
Examples for the last two marks in (c)

e.g. 1 $0.8^n<0.0016\Rightarrow n=29$ scores M0A0 (no method seen)

e.g. 2 $0.8^n<0.0016\Rightarrow n=28.85=29$ scores M0A0 (no method seen)

e.g. 3 $0.8^n<0.0016\Rightarrow n<\dfrac{\log\left(0.0016\right)}{\log(0.8)}\Rightarrow n=29$ scores M1A1 (condoning the incorrect inequality sign in intermediate work)

Note: Methods relying on trial and improvement or the equation solver will usually only score a maximum M1dM1M0A0 for setting up the equation or inequality to $A\times 0.8^n...B$

Question Number	Scheme	Marks
8(i)	$5\sin(3x+0.1)+2=0$ $\Rightarrow 5\sin(3x+0.1)=-2$ $\Rightarrow \sin(3x+0.1)=-\frac{2}{5}$	M1
	$\sin(3x+0.1)=-\frac{2}{5}$ $\Rightarrow 3x+0.1=\sin^{-1}\left(-\frac{2}{5}\right)$ $\Rightarrow x=\dfrac{\sin^{-1}\left(-\frac{2}{5}\right)-0.1}{3}$	dM1
	$x=-0.94,\ -0.17,\ \ 1.15,\ 1.92$	A1A1
		(4)
(ii)	$2\tan\theta\sin\theta=\cos\theta+5$ $\Rightarrow 2\sin^2\theta=\cos^2\theta+5\cos\theta$	M1
	$\Rightarrow 2(1-\cos^2\theta)=\cos^2\theta+5\cos\theta$	M1
	$\Rightarrow 3\cos^2\theta+5\cos\theta-2=0$	A1
	$\cos\theta=\frac{1}{3}(,-2)$ $\Rightarrow \theta=\cos^{-1}\left(\frac{1}{3}\right)=...$	M1
	$(\theta=)\ 70.5°,\ 289.5°$	A1
		(5)
		Total 9

Answers with no working score 0 marks

(i)

M1 For ±2 and then dividing by 5 to reach $\sin(3x+0.1)=\pm\frac{2}{5}$ oe

May be implied by $3x+0.1=\pm0.411...$ or allow a different variable to be used such that $y=3x+0.1 \Rightarrow \sin(y)=\pm\frac{2}{5}$.

Condone $3x+0.1=\pm23.57...$ if they have incorrectly worked in degrees for this mark only.

dM1 Correct strategy for finding x. Allow $x=\dfrac{\pm 2\pi n+\sin^{-1}\left(\pm\frac{2}{5}\right)\pm 0.1}{3}$ or

e.g. $x=\dfrac{\pi-\sin^{-1}\left(\pm\frac{2}{5}\right)\pm 0.1}{3}$ May be implied by a correct angle, but they must have proceeded as far as $\sin(3x+0.1)=-\frac{2}{5}$ before achieving an angle.

Must be working in radians OR entirely in degrees (if the 0.1 radians is converted first)
It is dependent on the first method mark.

A1 Two of awrt $-0.94, -0.17, 1.15, 1.92$. Must be in radians.

A1 All of awrt $-0.94, -0.17, 1.15, 1.92$ and no extras in range

Beware of $5\sin(3x+0.1)=-2 \Rightarrow 15\sin x+0.5=-2 \Rightarrow x=-0.17$ **which scores 0 marks**

Note: There are other credit worthy methods such as squaring $5\sin(3x+0.1)=-2$ and using $\pm\sin^2\theta=\pm 1\pm\cos^2\theta$, then solving a quadratic in cos θ. For the first M1 they would have to proceed as far $3x+0.1=...$ but allow the mark if slips in rearranging are made.

(ii)

M1 For using $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ and attempting to multiply by cos θ (may even be $\cos^2\theta$ or higher which is acceptable). Look for the denominator being removed from $\dfrac{\sin^2\theta}{\cos\theta}$ and multiplying at least one other term by cos θ (or $\cos^2\theta$ etc).

M1 Attempts to use $\pm\sin^2\theta=\pm 1\pm\cos^2\theta$ and proceeds to an equation in $\cos\theta$ only or $\sin\theta$ only

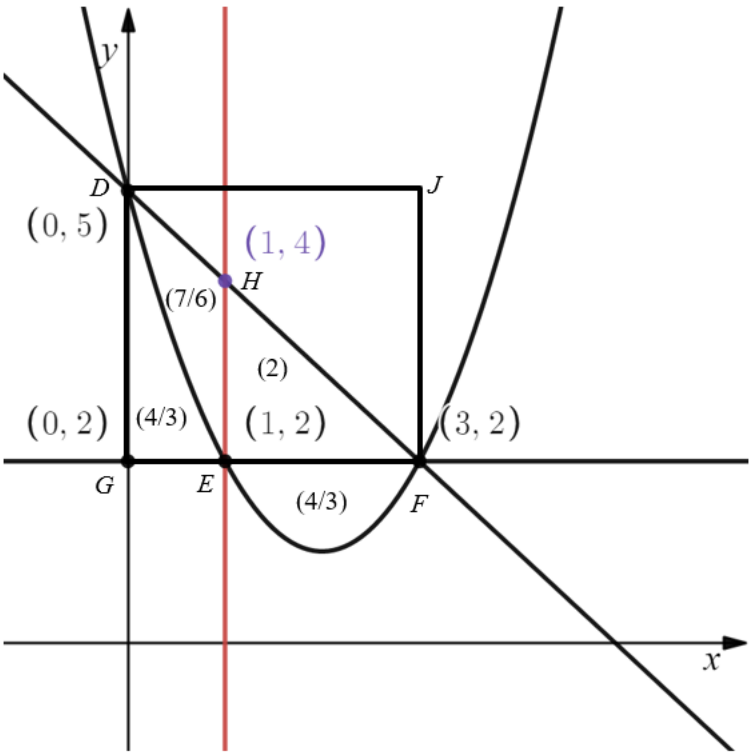
A1 $3\cos^2\theta+5\cos\theta-2\ (=0)$. Not all need to be on the same side of the equation and condone the omission of = 0 if all on one side. Condone poor notation for $\cos^2\theta$ eg $\cos\theta^2$

M1 Solves their 3TQ in cos θ and takes inverse cos of one of their roots to obtain at least one value for θ. As a minimum, expect to see the root(s) to their 3TQ before proceeding to an angle which may need to be checked. Condone their angle to be in radians eg typically awrt 1.23 or awrt 5.05

A1 awrt 70.5, awrt 289.5 and no others in the range. Must be in degrees not radians.

Question Number	Scheme	Marks
9(a)	$(0, 5)$	B1
		(1)
(b)	$x^2 - 4x + 5 = 2 \Rightarrow x^2 - 4x + 3 = 0$ $\Rightarrow x = ...$	M1
	$x(E) = 1,\ \ x(F) = 3$	A1
		(2)
(c)	Area $R_1 = \displaystyle\int_0^1 \left(x^2 - 4x + 5 - 2\right)\mathrm{d}x = \left[\frac{x^3}{3} - 2x^2 + 3x\right]_0^1 = \frac{1}{3} - 2 + 3 - 0 = \frac{4}{3}$	M1A1
	Area $R_2 = \dfrac{1}{2}\times 3\times 3 - "\dfrac{4}{3}" = \dfrac{19}{6}$ or $\displaystyle\int_0^1 \left(5 - x - \left(x^2 - 4x + 5\right)\right)\mathrm{d}x + \frac{1}{2}\times 2\times 2 = \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^1 + 2 = \frac{3}{2} - \frac{1}{3} + 2 = \frac{7}{6} + 2 = \frac{19}{6}$ or $\displaystyle\frac{1}{2}\times 3\times 3 - \int_0^1 \left(x^2 - 4x + 5 - 2\right)\mathrm{d}x = \frac{9}{2} - \left[\frac{x^3}{3} - 2x^2 + 3x\right]_0^1 = \frac{9}{2} - \left(\left(\frac{1}{3} - 2 + 3\right) - 0\right) = \frac{19}{6}$	M1A1
	$\dfrac{\text{area of } R_1}{\text{area of } R_2} = \dfrac{\frac{4}{3}}{\frac{19}{6}} = \dfrac{8}{19}$	A1
		(5)
		Total 8

<u>Diagram for reference:</u>



(a) Mark (a) and (b) together

B1 Correct **coordinates**. Allow to be written as $x=0,\ y=5$ (check the diagram as well)

5 on its own is B0

(b)

M1 Sets $C = l$ and solves for x. Expect to see the 3TQ $x^2-4x+3\ (=0)$ oe (terms do not need to be on the same side) before solving the quadratic by factorising, quadratic formula, completing the square or via a calculator to find a value for x.

A1 Correct values following a **correct method**. Condone x = 1, 3 and also allow if the correct y coordinates are also given e.g. $(1,\ 2)$ and $(3,\ 2)$. Condone if E and F are labelled the wrong way round and check the diagram for the coordinates. If there is a contradiction between Figure 3 and the coordinates stated in the main body of the work then the main body of the work takes precedence. isw after the correct x coordinates are found.

Condone e.g. $E=1, F=3$

Answers only scores M0A0 (Minimum seen must be $x^2-4x+5=2$ or x^2-4x+3 oe)

(c) They are many different strategies for finding the area of R_1 and R_2 so check solutions carefully for alternative credit worthy methods

M1 Fully correct strategy for the area of R_1

Attempting the area under curve C – area under the line $y=2$ between 0 and "1" Score for either

- attempting to integrate $C-l$: $\int_0^1 \left(x^2-4x+5-2\right)\mathrm{d}x$ oe
- attempting to integrate C $\int_0^1 \left(x^2-4x+5\right)\mathrm{d}x$ and subtracting the area of the rectangle

Condone slips in their working but expect to see the power raised by one on at least one of their terms and either substituting in the limits 0 and "1" (or subtracting "2" for the area of the rectangle). Must be the limits based on their part (b) answer.

Their integrated expression must be seen with the limits or an attempt to substitute in and subtract either way round – **the method mark cannot be scored directly from using a calculator.** Condone spurious notation.

A1 $\frac{4}{3}$ or exact equivalent correct area of R_1 provided M1 has been scored.

M1 **Fully correct strategy for the area of R_2.**

Note that If R_1 is used to find the area of R_2 then R_1 must have been found from a correct strategy.

There are MANY possible strategies including:

- $GDF-"R_1"$
- $EDH+EHF$
- Finding the equation of the line through DF $y=5-x$ and attempting to integrate $DF-C$ between 0 and "3", then finding the area bounded by l and the curve between "1" and "3" and subtracting this from the total
- $DJFG-"R_1"-DJF$

In all cases you need to score for the overall strategy to find R_2 so condone slips in their working and spurious notation but if the method is unclear then the required areas must be correct for their coordinates in (a) and (b). If integration is used on a new area (not R_1) then expect to see the power raised by one on at least one of their

terms and the integrated expression must be seen with the limits or attempting to substitute in and subtracting either way round. Must be the limits based on their part (b) answer.
If there are multiple attempts then score the most complete attempt.

A1 $\frac{19}{6}$ or exact equivalent correct area of R_2 provided M1 has been scored

A1 $\frac{8}{19}$ cao and all previous marks have been scored

Question Number	Scheme	Marks
10	Considers another relevant case for n e.g. $n=3k-1$ or $3k+1$ or $3k+2$ For example $n = 3k + 1$ $\Rightarrow 2n^2+n+1=2(3k+1)^2+(3k+1)+1=18k^2+15k+4$	M1
	$=3(6k^2+5k+1)+1$ which is not divisible by 3	A1
	Considers a third relevant case to complete the 3 cases for n e.g. one of $n=3k-1$ or $3k+1$ or $3k+2$ For example $n = 3k - 1$ $\Rightarrow 2n^2+n+1=2(3k-1)^2+(3k-1)+1=18k^2-9k+2$	dM1
	$=3(6k^2-3k)+2$ which is not divisible by 3 Hence $2n^2+n+1$ is not divisible by 3	A1
		(4)
		Total 4

Condone the use of another letter or even n for the different cases for the first 3 marks.
Do not withhold accuracy marks for any errors which are not part of the main body of the work.
Look at the whole solution and score for a correct case first (which may appear later) so that the first two marks M1A1 can be awarded.

M1 Considers another relevant case for n by attempting to substitute in their expression in k, multiplies out brackets and collects terms. Condone slips.
May attempt eg $3k+4$ or any other multiples of those given which is acceptable.

A1 Completes the process with no errors to show their expression is not divisible by 3. Usually this is by factorising part of the expression, but may identify a particular term which is not divisible by 3.

dM1 Considers a third relevant case which is different to the given case and not an equivalent one to their first relevant case to complete the 3 cases for n by attempting to substitute the final different expression in k, multiplies out brackets and collects terms. Condone slips.

A1 Completes the process with no errors to show their 3rd expression is not divisible by 3 (usually by factorising, but may identify a particular term which is not divisible by 3) and concludes the proof with a statement e.g. hence ($2n^2+n+1$) not divisible by 3. Alternatively, they may write a statement/preamble at the beginning and the proof can be ended with a tick/QED oe
Must have been in terms of k for all cases.

The table below may help when checking for accuracy of quadratics

Case	$2n^2+n+1$	Possible factorisation of expression
$3k-2$	$18k^2-21k+7$	$=3(6k^2-7k+2)+1$
$3k-1$	$18k^2-9k+2$	$=3(6k^2-3k)+2$
$3k+1$	$18k^2+15k+4$	$=3(6k^2+5k+1)+1$
$3k+2$	$18k^2+27k+11$	$=3(6k^2+9k+3)+2$
$3k+4$	$18k^2+51k+37$	$=3(6k^2+17k+12)+1$

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